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## Numerical Simulating for Turbulent Heat Transfer of Thermoacoustic Cooler

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### Abstract

This article discussed thermoacoustic refrigeration with method of numerical simulation. Provided theory model of numerical simulation firstly, and simulated a thermoacoustic cooler model. Researches focus on temporal and spatial variation of parameters including temperature, velocity, pressure, density and turbulent intensity. Treated the heat exchangers and resonator as porous media, with 15atm pressure and 20Hz frequency, achieved the lowest temperature is 243K. Discussed the mechanism of these results at last, deem that the gas, at the impact of pressure wave, oscillated back and forth in pulse tube, urge the heat in right space transmitted outside via exchangers and resonator. These conclusions will take guiding function for the development of thermoacoustic cooler.

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*Keywords:* Thermoacoustic Refrigeration; Thermoacoustic Engine; Numerical Simulation; Porous Media

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### 1. Introduction

There are two kinds of thermoacoustic phenomenon, one is acoustic effect caused by heat, and the other is low temperature caused by acoustic wave. The former effect is mechanism that heat energy transformed into mechanical energy; the later is heat transport caused by acoustic wave, generate temperature difference. This effect is the realization of the transformation of mechanical energy into heat energy, can be applied in refrigeration.

As the development of thermoacoustic theory, the research on thermoacoustic engine has came into the stage of application. Thermoacoustic engine[1, 2] and thermoacoustic cooler[3, 4] has been in research,

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and part of them had been in use. Researches on thermoacoustic cooler with CFD method are mainly on the following aspects: Banjare[5] simulated the oscillating flow inside a pulse tube cryocooler, he got the conclusion that low frequency cause better cooling effect, and verify the reliability of CFD method in thermoacoustic study. Li Zhuopei[6] studied the pulse tube cooler driven by thermoacoustic engine with both numerical and experimental method. He discussed the influence of frequency, pressure ratio, charging pressure and regenerator length on the refrigeration performance. The lowest temperature of the cooler was expected to be 45K according to the simulation, and the optimization of the cooler is to enlarge the pressure ratio, to lower the frequency and increase the pressure. There are great number of researches about the detail flow and heat transfer around the stack: Tasnim[7] gave out the detail velocity vector and temperature field between two stacks of a simplified cooler drove by acoustic wave, and caught the spatial temperature difference of the fluid around the stacks during an acoustic vibration period. Florian Zink[8] simulated the both effects of thermoacoustic and thermoacoustic refrigerator. He got the oscillation frequency and the sound pressure amplitude of 614Hz and 4700Pa by his thermoacoustic engine model. Then, he observed the decrease of average temperature. Herman[9] investigated the heat transfer of the stacks and exchanger in thermoacoustic cooler. He holds that the distribution of temperature and heat flux around the stacks is nonlinear, and he divided the heat transfer in the gas near stacks into four thermodynamic processes. Marx[10] solved the compressible unsteady N-S equations by numerical method, quested the flow condition near the thermoacoustic engine stacks, got the nonlinear temperature oscillation which is unforeseen in linear thermoacoustic theory. He considers that it will get the best heat pump effect when the distance between stacks and heat exchangers equals the swing of gas particle. Zhang Xiaoqing[11] simulated the regenerator of thermoacoustic engine, investigated the conditions of flow and heat transfer near the stacks in a thermoacoustic cooler with the method of simple harmonic analysis. Huang Jing[12] did the research on temperature evolution near the stacks and time-averaged energy flow in one cycle based on compressible pressure correction method.

In this article, we will first give the theory on thermoacoustic refrigeration, then provide a model of thermoacoustic cooler, and discuss the mechanism of its refrigeration by the method of numerical simulation.

## 2. Mathematical Model of Thermoacoustic Cooling

Compressible unsteady standard k-ε theory model include 7 unknown parameters that is  $\langle p \rangle$ ,  $\langle \rho \rangle$ ,  $\langle T \rangle$ ,  $\langle u \rangle$ ,  $\langle v \rangle$ ,  $k$  and  $\varepsilon$ , and include 7 equations accordingly:

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial \langle \rho \rangle \langle u \rangle}{\partial x} + \frac{\partial \langle \rho \rangle \langle v \rangle}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial \langle \rho \rangle \langle u \rangle}{\partial t} + \frac{\partial (\langle \rho \rangle \langle u \rangle \langle u \rangle)}{\partial x} + \frac{\partial (\langle \rho \rangle \langle v \rangle \langle u \rangle)}{\partial y} = & -\frac{\partial \langle p \rangle}{\partial x} + \frac{\partial}{\partial x} \left[ \frac{4}{3} \mu_t \frac{\partial \langle u \rangle}{\partial x} - \frac{2}{3} \mu_t \frac{\partial \langle v \rangle}{\partial y} - \frac{2}{3} \langle \rho \rangle k \right] \\ & + \frac{\partial}{\partial y} \left[ \mu_t \left( \frac{\partial \langle v \rangle}{\partial x} + \frac{\partial \langle u \rangle}{\partial y} \right) \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial \langle \rho \rangle \langle v \rangle}{\partial t} + \frac{\partial (\langle \rho \rangle \langle u \rangle \langle v \rangle)}{\partial x} + \frac{\partial (\langle \rho \rangle \langle v \rangle \langle v \rangle)}{\partial y} = & -\frac{\partial \langle p \rangle}{\partial y} + \frac{\partial}{\partial x} \left[ \mu_t \left( \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right) \right] \\ & + \frac{\partial}{\partial y} \left[ -\frac{2}{3} \mu_t \frac{\partial \langle u \rangle}{\partial x} + \frac{4}{3} \mu_t \frac{\partial \langle v \rangle}{\partial y} - \frac{2}{3} \langle \rho \rangle k \right] \end{aligned} \quad (3)$$

$$\frac{\partial (\langle \rho \rangle \langle T \rangle)}{\partial t} + \frac{\partial (\langle \rho \rangle \langle u \rangle \langle T \rangle)}{\partial x} + \frac{\partial (\langle \rho \rangle \langle v \rangle \langle T \rangle)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \frac{k_f}{c_p} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \langle T \rangle}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \frac{k_f}{c_p} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \langle T \rangle}{\partial y} \right] + \langle S_T \rangle \quad (4)$$

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial(\rho k u)}{\partial x} + \frac{\partial(\rho k v)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + G_k + G_b - \rho \varepsilon - Y_M \quad (5)$$

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial(\rho \varepsilon u)}{\partial x} + \frac{\partial(\rho \varepsilon v)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (6)$$

$$\langle p \rangle = R \langle \rho \rangle \langle T \rangle \quad (7)$$

Because the exchangers and resonator are treated as porous media, we need tackle the source term as follow:

(1) Source term of momentum equations:

There are additional source terms in momentum equations of porous media, these source terms consist of two parts, the viscous loss term and the inertial loss term. In the two dimensional Cartesian coordinate system:

$$\begin{aligned} \langle S_x \rangle &= - \left( \frac{\mu}{\alpha} \langle u \rangle + C_2 \frac{1}{2} \rho |U| \langle v \rangle \right) \\ \langle S_y \rangle &= - \left( \frac{\mu}{\alpha} \langle v \rangle + C_2 \frac{1}{2} \rho |U| \langle u \rangle \right) \end{aligned} \quad (8)$$

In equation(8),  $S_x$  and  $S_y$  are momentum source terms at two direction,  $\mu$  is molecular viscosity,  $|U|$  is the module of velocity. For the simple porous media, we can take the  $D_{ij}$  and  $C_{ij}$  as diagonal matrix  $1/\alpha$  and  $C_2$  ( $\alpha$  is permeability, and  $C_2$  is the inertial resistance factor).

(2) Energy equation of porous media:

We still solve the standard energy transport equation in porous media regions, but modify the conduction flux and the transient terms. In the energy equation of porous media, it assumed that the fluid and solid have same temperature when use one-temperature model, in unsteady term, considered the different density of fluid and solid in porous area. The conduction flux uses the effective conductive factor  $k_{fs}$  which calculated by averaging the thermo-conductivity of fluid and solid:

$$k_{fs} = \gamma k_f + (1 - \gamma) k_s \quad (9)$$

Where  $k_f$  is thermoconductivity of fluid (include the impact of turbulence  $k_t$ ),  $k_s$  is thermoconductivity of solid,  $\gamma$  is porosity.

### 3. Physics Model and Numerical Method

Thermoacoustic cooler model discussed in this article composed of a close ended resonance tube, cold heat exchanger (CHX) and hot heat exchanger (HHX), and resonator, as figure1. It pumps heat and bring temperature difference once supply sound source at left surface. The length and radius of the tube is 170cm and 6cm separately, between  $x=30\text{cm}$  and  $x=40\text{cm}$  is HHX, and CHX is between  $x=60\text{cm}$  and  $x=70\text{cm}$ , between the two heat exchangers is resonator which used in heat transmission. The heat exchangers and resonator are critical components of the cooler, which selected metallic material in general for its excellent conduction. Researchers likely use metallic parallel stacks in existing research, but in this article, we treat it as porous media.

In stead of traditional Freon, we padded the inert gas as working substance in the closed tube, so it environment friendly. To get better cooling effect, the pressure is higher than normal atmosphere[6], it is 15atm. We supply a sinusoidal pressure wave at the left surface:

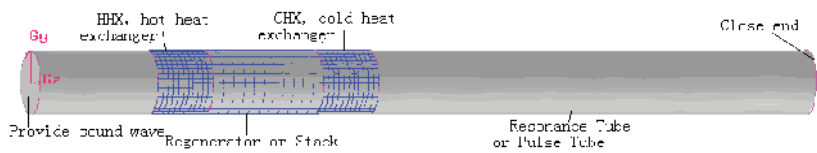


Figure 1. Physical model of thermoacoustic cooler

$$p = A\sin(2\pi ft) + B$$

Where  $A = 506625 \text{ Pa}$ , stand for the amplitude of the wave is  $5\text{atm}$ ; and  $f = 20 \text{ Hz}$  is the frequency of oscillation (the low frequency makes better cooling effect[5,6]). The wall is adiabatic.

Though exchangers and resonator treated as porous material in the cooler, and this avoid complex treatment than use parallel stacks, the coefficient of viscous loss and inertial loss should be set carefully for the directivity flow in the heat exchange part. The porosity is  $0.69$ , and material is copper with the heat conductivity  $387.6 \text{ w/m}\cdot\text{k}$ , specific heat  $381\text{j/kg}\cdot\text{k}$ , and density  $8978\text{kg/m}^3$ . Physical property of argon will be in table 1:

Table 1. Physical property of argon

| Density(kg/m3)  | Cp(j/kg·k) | Heat conductivity(w/m·k) | Viscosity(kg/m·s) | Molecular Weight(kg/kgmol) |
|-----------------|------------|--------------------------|-------------------|----------------------------|
| Ideal gas model | 520.64     | 0.0158                   | 2.125e-05         | 39.948                     |

This work was simplified as two dimensional, unsteady, compressible problems in our numerical model. We apply pressure based solver, first-order upwind scheme, and PISO arithmetic. And the under-relaxation factor would be small in order to strengthen the stability of solution. The initial velocity is  $0\text{m/s}$ , initial temperature is  $300\text{K}$  in the field, time step is  $0.005\text{s}$ . To detect the evolution of temperature on HHX and CHX, we set two monitoring point at  $x = 35\text{cm}$  and  $65\text{cm}$  along axis. And put the horizontal monitoring surface  $y = 6\text{cm}$  to detect distribution of temperature and other parameters along axis.

4. Results and discussion

Firstly, to verify the validity of our inlet pressure boundary condition by UDF of Fluent6.3, we set detector at the left wall of the model. Through figure2, we can see that there exists sinusoidal pressure pulsating at the left pressure boundary, and this is sound wave, with the amplitude of  $5\text{atm}$  and frequency  $20\text{Hz}$ , swing regularly around average pressure in the model. This low frequency and high amplitude will promote the temperature difference.

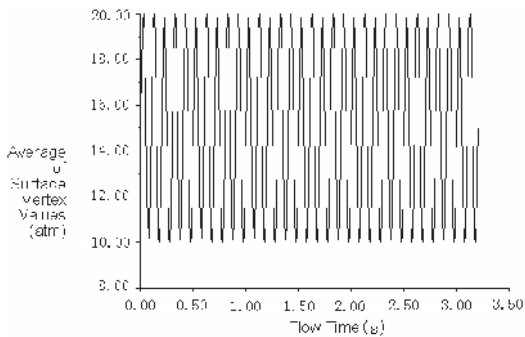


Figure 1. Sinusoidal pressure wave

Under these conditions, we take the unsteady numerical calculation. At the initial stage, when calculation time  $t = 10\text{s}$ , the result shows temperature varying with time on two heat exchangers as figure 3. Two pictures in figure 3 are detectors at  $x = 0.35\text{cm}$  and  $x = 0.65\text{cm}$  separately, i.e. at the hot heat exchanger and the cold heat exchanger. Result shows that although two curves fluctuated more or less, the average temperatures are developed toward low and high separately.

We can read from the two pictures, at  $t = 10\text{s}$ , the average temperature on the HHX rise above  $325\text{K}$ , ascended by  $25\text{K}$ ; and temperature on CHX

demonstrated the tendency of decline obviously, but maybe as the large left space and ample heat energy of this model, the extent of temperature drop is not so distinct at the initial stage, only about 1K. The fact that thermoacoustic induce low temperature is verified primarily.

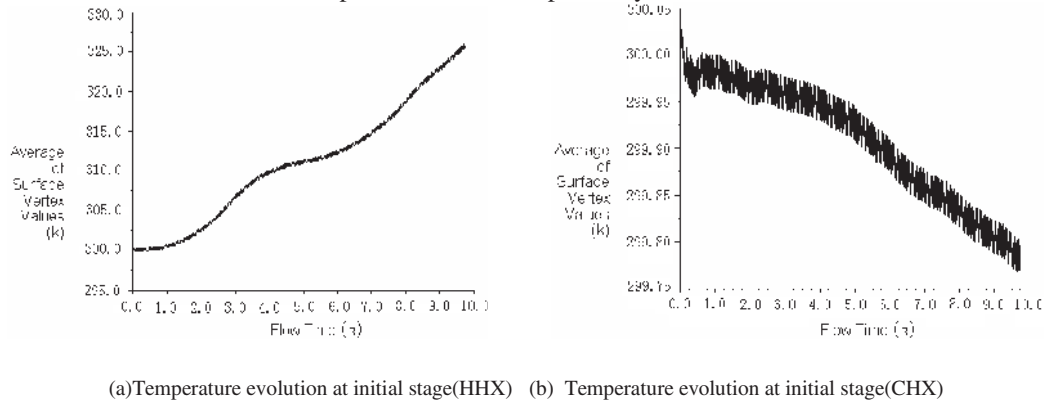


Figure 2. Temperature difference emerged at initial stage(10s)

As the tendency displayed above, let the calculation proceed. When  $t=21$ s, we examined the contours of space temperature distribute in the model, and got the result as figure4 displayed. From figure4, we can see that there present distinct temperature difference in the whole model. Temperature in the right space drop clearly, and the minimum value reached 264K; from right to left, the more closer to HHX, the higher of temperature, and it layered clearly. At HHX, temperature reached its maximum, about 491K. This phenomenon is our thermoacoustic refrigeration, the device apply this mechanism to refrigerator will be new type thermoacoustic cooler. At the drive of left-hand end pressure wave, the pressure, velocity and density interact with each other, cause the heat exchangers pump heat from interior(right) to outside(left), then bring about the outcome of temperature difference in the field.

We can speculate from this variation tendency, as long as the calculation continue, the value of temperature difference maybe enlarge. So, we did it, and the results show that at the time of  $t=55$ s it reached a relative steady state. At this stage, parameters rested, and changed little with time and space. And this maybe the cooling limit of this model. These results will be the following.

From the scale in figure5 we can see that the maximum temperature near HHX was reached 525K, and the minimum value in the right pulse tube was lower to 243K. In resonator and at heat exchangers, temperature gradient is visible, and the layered temperature reflected the process that heat was pumped out at the effect of pressure pulse. Parameters distributed along the model axis were provided by figure6. It is easy to see that temperature distribution along axis, which is similar as the discipline expressed in figure5, reached its maximum at HHX, and minimum on the right end of pulse tube, but keep the initial 300K at the left inlet boundary because we set it constant. Pressure distribution was divided into high pressure and low pressure zone two parts which separated by heat exchangers. The minimum value of pressure is about 15.3atm in pulse tube, and maximum pressure nearly 20atm, heat exchangers displayed as the transition zone. The rule of density distribution is contrary to temperature distribution on the whole, low density at high temperature and high density at low temperature. In figure6(d), the velocity value nearly zero on the right of the tube, and it diminished as close to right-hand end, but toward left along axis, velocity increase rapidly, and jumped when it depart from HHX, the peek value almost 17m/s. It decreased when close to left surface. Turbulent intensity distributed as velocity, large velocity with large turbulent intensity, vice versa.

To certify velocity distribution and examine the flow direction, figure7 demonstrated velocity vector in detail, as stated before, velocity mainly scattered at left part. Working gas inversed when it flow left

and encountered left wall, and fluid reciprocated in the pulse tube which cause the heat in right resonance tube transported to HHX on the left.



Figure 3. Distribution of temperature in the model at t=21s



Figure 4. Distribution of temperature in the model at t=55s

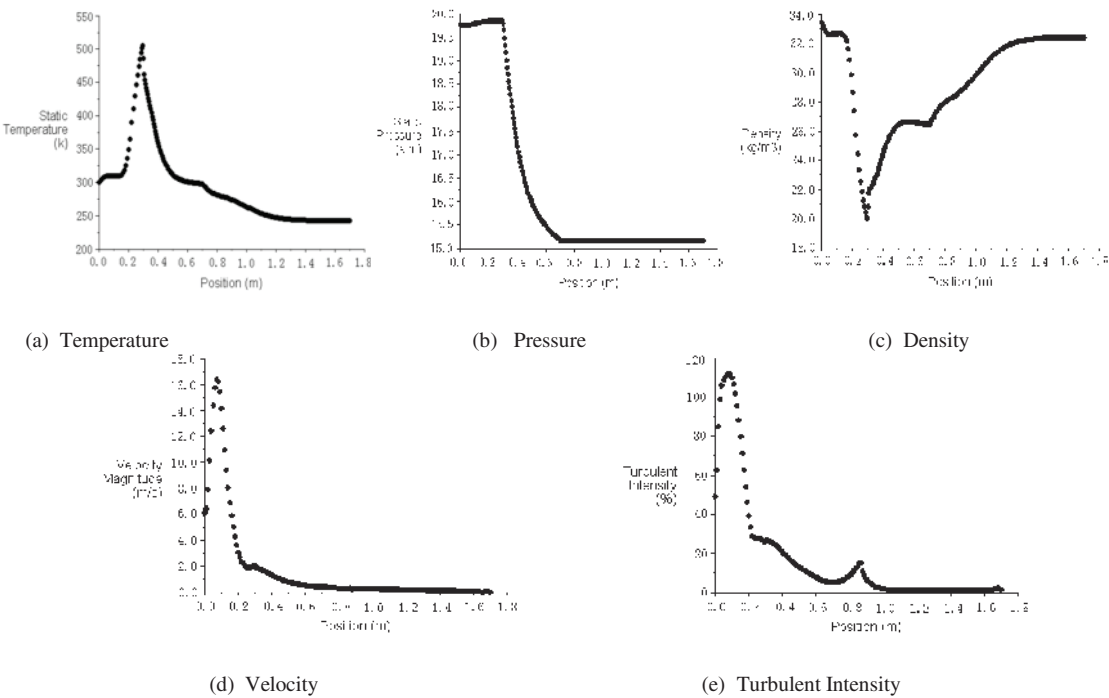


Figure 5. Distribution of parameters along axis (y=6cm)

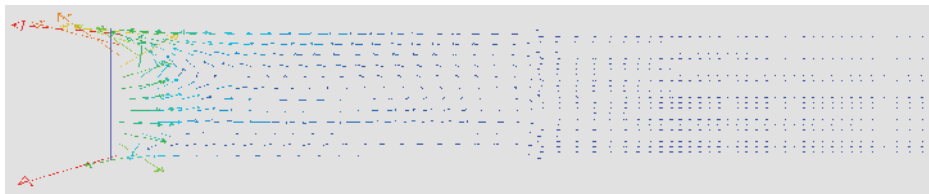


Figure 6. Distribution of velocity vectors at left part of the model

## 5. Conclusion

This article discussed thermoacoustic refrigeration with method of numerical simulation. We first sum up the current research situation and conclusion briefly, then provide theory model of numerical simulation, and at last, a thermoacoustic cooler model was declared and simulated. The research focus on physical properties such as temperature, velocity, pressure, density and turbulent intensity, and primarily studied the temporal and spatial variation of parameters. We treated the heat exchangers and resonator as porous area, with 15atm pressure and 20Hz frequency, achieved the lowest temperature is 243K; discussed the mechanism of these results, hold that the gas, at the impact of pressure wave, oscillated back and forth in pulse tube, compressed and expanded repeatedly, urge the heat in right space transmitted outside via exchangers and resonator. These conclusions will take guiding function for the development of thermoacoustic cooler.

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